Potenzen, Wurzeln, Wurzelgleichungen P10-Lsg

(1) a) $\sqrt[a]{\left(\frac{n^{x+a}}{n^x}\right)^c} = \left(\left(n^a\right)^c\right)^{\frac{1}{a}} = n^{a\cdot c\cdot\frac{1}{a}} = \underline{\underline{n^c}}$

b) $\frac{2}{a^{-3}} + \left(a^{\frac{1}{4}}\right)^{-\frac{4}{3}} = 2a^3 + a^{-\frac{1}{3}} \left(= 2a^3 + \frac{1}{\sqrt[3]{a}}\right)$

---

(2) b) $\left(\frac{6a^2\cdot y^{-3}}{10x^2b^{-3}}\right)^3 \cdot \left(\frac{3ay^{-2}}{5x^2b^{-2}}\right)^{-3} = \frac{6^3a^6y^{-9}\cdot 3^{-3}a^{-3}y^6}{10^3x^6b^{-9}\cdot 5^{-3}x^{-6}b^6}$

$= \frac{\cancel{2^3}\cancel{3^3}\cdot\cancel{3^{-3}}\cdot a^3y^{-3}}{\cancel{2^3}\cdot\cancel{5^{-3}}\cdot\cancel{5^{-3}}\cdot b^{-3}} = \underline{\underline{\left(\frac{ab}{y}\right)^3}}$

a) $(a+b)^{2a-3+4a+6} = (a+b)^{6a+3}$

c) $\left(\frac{\cancel{13}^{1}a^3\cancel{x^4}}{\cancel{10}_{1}\cancel{x^4}}\cdot\frac{\cancel{3}^{3}\cancel{2}x^3\cancel{y^3}}{\cancel{26}_{2}\cancel{a^4}}\right) - \left(\frac{\cancel{24}^{3}a^3\cancel{x^4}}{\cancel{13}_{1}\cancel{x^4}}\cdot\frac{\cancel{26}^{2}\cancel{x^3}}{\cancel{8}_{1}}\right) = \frac{3}{2}a^3x^3 - \frac{6}{1}a^3x^3$

$= -\frac{9}{2}a^3x^3 = -4\frac{1}{2}a^3x^3$

---

(3) $\left[\frac{\cancel{6}\cancel{(x^2-2ab)}\cdot 4a}{\cancel{(x^2-2ab)}\cdot\cancel{2}6}\right]^3 + \frac{4^3a^3}{b^3} = 64\left(\frac{a}{b}\right)^3 + 64\left(\frac{a}{b}\right)^3 = 128\left(\frac{a}{b}\right)^3$

$= 128\,\frac{a^3}{b^3}$

(4) a) $\frac{\cancel{a}\cancel{(a-1)}}{\cancel{a}\cancel{(a+1)}} \cdot \frac{\cancel{(a+1)}(a-2)}{(a+1)\cancel{(a-1)}} \cdot \frac{(a-2)^2}{(a+1)^2} = \left(\frac{a-2}{a+1}\right)^3 = \left[\frac{(a-2)^3}{(a+1)^3}\right]$

b) $\frac{n^{\cancel{5}}y^{\cancel{3}}x^{\cancel{3}}}{\cancel{n^4}y\,x^{\cancel{5}\,2}} + \frac{2n^{\cancel{3}}y^{\cancel{5}}x^{\cancel{6}\,2}}{\cancel{x^4}\cancel{n^2}\cancel{y^5}} - \frac{3n^{\cancel{7}\,6}y^{\cancel{11}}x^{\cancel{12}}}{\cancel{n^3}\cancel{y^{10}}\cancel{x^{11}}} = \frac{ny}{x^2} + \frac{2x^2y^{\cancel{2}\,1}}{1} - \frac{3nxy^{\cancel{3}\,2}}{1}$

$\frac{ny}{x^2} + \frac{2nx^4y}{x^2} - \frac{3nx^3y}{x} = \frac{ny + 2nyx^4 - 3nyx^3}{x^2} = ny\frac{(1+2x^4-3x^3)}{x^2}$

(5) a) $4\sqrt[a]{\frac{b^3}{\cancel{b^2}c}} - 3\sqrt[a]{\frac{b}{c}} + 2\sqrt[a]{\frac{b}{c}} = 3\sqrt[a]{\frac{b}{c}} - 3\left(\frac{b}{c}\right)^{\frac{1}{a}}$

b) $\frac{x^{\frac{a+3b}{a-b}}}{x^{\frac{4b}{a-b}}} + \frac{x^{\frac{3a+2b}{a+b}}}{x^{\frac{2a+b}{a+b}}} = x^{\frac{a+3b-4b}{a-b}} + x^{\frac{3a+2b-2a-b}{a+b}}$

$= x^{\frac{a-b}{a-b}} + x^{\frac{a+b}{a+b}} = x^1 + x^1 = \underline{\underline{2x}}$

(6) $\sqrt{5x+70} + \sqrt{6-x} = 4\sqrt{x+7}$ $\quad |^2 \quad \mathbb{D} = \{x \mid -4\frac{2}{3} \le x \le 6\}_{\mathbb{R}}$

$x \ge -4\frac{2}{3}$ $\quad x \le 6$ $\quad x \ge -7$

$5x + 70 + 2\sqrt{5x+70}\sqrt{6-x} + 6 - x = 16(x+7)$

$14x + 76 + 2\sqrt{5x+70}\sqrt{6-x} = 16x + 112 \quad | -14x - 76 \quad | \to :2$

$\sqrt{5x+70}\sqrt{6-x} = x + 18 \quad |^2$

$(5x+70)(6-x) = x^2 + 36x + 324$

$30x - 5x^2 + 420 - 70x = x^2 + 36x + 324$

$16x^2 + 16x - 96 = 0 \quad (|:16 \Rightarrow x^2 + x - 6 = 0)$

$x_1 = 2 \; ; \; x_2 = -3$

$\mathbb{L} = \{-3 / 2\}$